SUPER!!

Supercorrection Four-Point Form Name:

#2 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 3 /4 Supercorrection 4 /4 My error in this was simply not thinking logically about the problem, and definitely checking my answer. I should have realized I could change that last digit of 5 to 49, the calculater just automatically rounds. I needed to develop wisdom beyond my calculater. Next time I will halt myself and really ponder whether I have the smallest answer possible, or if there's a catch.

Correct solution:

Find the smallest value of x for which $3^x \geq 1{,}000{,}000$

$3^x \geq 1{,}000{,}000$ Irrational number.

$\log 3^x = \log 1000000$

$\frac{x \log 3}{\log 3} = \frac{\log 1000000}{\log 3}$

$x = 12.57541965$ This is what I previously got on a calculater. In reality that 5 at the end can be written as 49, because 49 is rounded to 5 on a calculater. If you plug in the two solutions: $3^{12.57541965} = 1000000.005$

$3^{12.575419649} = 1000000.004$ which is smaller! *

* In conclusion

$\boxed{x = 12.575419649}$ there's still a smaller number!

#3 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 3 /4 Supercorrection 4 /4 I was very very close in solving this problem. I believe my error was not only rounding, but rounding too early. I ran into a similar problem on p. 505, #7, which actually involved the same formula: $B = B_0 e^{kt}$ when solving this problem, which also had 400 as its bacteria at $t=0$, I should have remembered how crucial rounding is. Keeping my numbers as they were, it's now easy to find the correct answer. I need to remember the later you round, the more percise my answer will be.

Good!

Correct solution:

What I had before

$B = B_0 e^{kt}$

$400 = t(0)$

$900 = 4\ h.$

$400 = 400 e^{k(0)}$

$B_0 = 400$

$\frac{900}{400} = \frac{400 e^{k(4)}}{400}$

$\left(\frac{9}{4} = e^{4k}\right) \ln$

$\frac{\ln 9/4}{4} = \frac{4k}{4}$

$\boxed{k = \frac{\ln 9/4}{4}}$

or $k = \frac{\ln 2.25}{4}$

NOW instead of trying to find this value and round, leave it as it is and plug it in.

1900 bacteria = 12 hours?

$B = B_0 e^{kt}$

$B = 400 e^{\left(\frac{\ln 2.25}{4}\right)12}$

$\left(\frac{\ln 2.25}{4}\right)12 = 2.432790649$

$B = 400 e^{2.432790649}$

$\boxed{B \approx 4556}$ so, $\boxed{NO}$

At 12 hours, there is 4556 bacteria which $\neq 1900$

Supercorrection Four-Point Form Name:

#5 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 3/4 Supercorrection 4/4 I actually thought I was decently close on this one as well. Looking back, I realize this was a rather simple problem, and I'd done it many times before. The equation given: $y = 2^x$ is a exponential equation. To change it to y form, I just need to reverse it back to the other form of the equation. On p. 474, under "The deffinition of a Logarithmic Function" it states how normally to get the inverse, we would just *

Correct solution: Write the equation of the inverse of $y = 2^x$ in y = form.

*swap x and y to get $y = a^x \longrightarrow x = a^y$. That is what I did, but on the next page is the deffinition. $y = \log_a(x)$ is equivalent to $x = a^y$, I should have known how to do this also because it was on the previous Quiz. Next time will just try to work it all the way through + think a little more.

I had: $y = 2^x$ then I swaped the x any y and got $x = 2^y$. Here is where I needed to use the deffinition and change this exponential equation into log form.

Good!

$x = 2^y$ is equivalent to: $\boxed{y = \log_2 x}$

#6 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 2/4 Supercorrection 4/4 I had gotten the first problem right, which dealt with properties of logarithms, but I must have not referred to p. 495 + p. 487-489. These contain all the neccisary rules for manipulating the equation in question. As noted below, I found exactly what rule I could apply + showed it. My error was not fully understand/Reviewing + Memorizing these properties. They are essential in dealing with logs. To fix this, I referred back to the text book, and found what I didn't know before.

Correct solution:

Use properties to write expression as sum, difference, and/or multiple of logs.

$\log \dfrac{100\sqrt{y}}{x^2}$

prop 6.

$= \log 100\sqrt{y} - \log x^2 \quad \left(\log \frac{M}{N} = \log M - \log N\right)$

$= \log 100 + \log\sqrt{y} \ (\log M \cdot N = \log M + N) - \log x^2$ prop. 3

$\log_{10} 100 = 2$

$2 + \log\sqrt{y} - \log x^2$

$\log \sqrt[2]{y} = 1/2 \log y \quad (\log \sqrt[a]{y} = 1/a \log \sqrt{y})$

(rule of exponents)

$\log x^2 = 2 \log x \ (\log_a M^p = p \cdot \log_a M)$

Prop. 7

Nice!

In conclusion:

$\boxed{2 + 1/2 \log y - 2 \log x}$

Supercorrection Four-Point Form Name:____________

9 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 0 /4 Supercorrection 4 /4 To figure this out, $a>b$, I know that b can't be negative, because the base of a logarithm: $\log_b a$ (b) can't be negative. You also know that $\log_b a$, if it's <0, has to be a negative #. The only way to turn a fraction to a negative number, is to raise it to a negative exponent. Now there will be a little guess and check involved, but one of the easiest fractions to manipulate is $\frac{1}{2}$. To demonstrate this

Correct solution: Select values for a and b such that $a>b$, such that $\log_b a < 0$. Justify.

$a>b$ — no negative base ($b \neq$ negative)

$\log_b a < 0$

$\log_{1/2} 4 = y$ $\quad 4 = 2^2$

$\frac{1}{2}^y = (2)^2$

$\frac{1}{2}^{-2} = 4$

$y = -2$ which is <0.

$4 = a$

$2 = b$

$\log_2 4 = 0$ $\quad 2 \neq 0$ $\quad \boxed{a=4,\ b=\frac{1}{2}} = a>b$

In an example, to prove to whole #'s don't work: $\log_2 4$ is equal to 2 which is $\not<0$. So now I have $\log_{1/2} x = y$ $(y<0)$ after trying a few #'s, 4 is chosen.

$\log_{1/2} 4 = y$

$\frac{1}{2}^y = (2)^2$ (another way to write 4)

$y = -2$ $\quad \frac{1}{2}^{-2} = 4$

10 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 1 /4 Supercorrection 4 /4 I had gotten the first step in changing $\log_a x + \log_a (x-2) = \log_a (x+4)$ previously by using property #5, but after that I was rather clueless. I must not have been thinking clearly & certainly did not remember that I could base log both sides of an equation to cancel out the same log on both sides. Once realizing that was an option, the problem was quite simple from there, the equation turning into a quadratic that could easily be factored. An important piece of this problem is checking your answers. I had gotten two, but upon plugging them in realized one,

Correct solution:

$\log_a x + \log_a (x-2) = \log_a (x+4)$

$= \log_a x^2 - 2x = \log_a (x+4)$ (Rule: #5 [p. 487])

Then you can raise both sides of the equation to a, to cancel out the logs. (base-a log both sides, kinda like on p. 495)

$a^{\log_a x^2 - 2x} = a^{\log_a (x+4)}$

Then I'm ~~just left~~ with

$x^2 - 2x = \cancel{x+4}$ ← was useless.

$\cancel{x-4}$

$x^2 - 3x - 4 = 0$ (Now could use quadratic or factor to do this)

$(x+1)(x-4)$ $\quad x = -1$ or 4

Now plug in. $\log_a (-1)$ → already against the rules of logs.

To try 4 Now:

Good!

$\log_a 4 + \log_a (4-2) = \log_a (4+4)$

$\log_a (4 \cdot 2) = \log_a 8$

$\log_a 8 = \log_a 8$

$\boxed{x=4}$

Supercorrection Four-Point Form Name:

\# 7 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 2 /4 Supercorrection 4 /4

Once again, this was a mental error in not taking more time to look over the rules and properties of logarithms and really understand them inside and out and be able to apply them in any order and to any problem. Throughout this ~~test~~ its been a problem for me, and my biggest fatal error. Now that I've taken the time too look back + carefully study the laws, the manipulations seem quite simple.

Correct solution:

$\ln 3 + 1/3 \ln(4-x^2) - \ln x$

$\ln 3 + \sqrt[3]{4-x^2} - x$ Problem.

- $\ln$ is present in every part of this equation [so I can take it aside:
$\ln | 3 + 1/3(4-x^2) - x$] This is actually a problem.

I can then take the cube root of $4-x^2$ by re-writing it as

The cube wasn't applied to the $-x$, so I just leave it. Then, referring back to property 6.

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$. So...

$\boxed{\ln \frac{3 + \sqrt[3]{4-x^2}}{x}}$ is the single quantity of $\ln 3 + 1/3\ln(4-x^2) - \ln x$

Check answer on the calculator by using a value of x....

\# 8 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 3 /4 Supercorrection 4 /4

The range is a whole other problem. You can't raise e to anything to get zero, so it must be greater than 0. In finding the Range, its your output, but in the Inverse equation, its your input. So, $2x-7>0$ (input of $f^{-1}(x)$, has to be bigger then zero) Then solve

$+7 \quad +7$

$\frac{2x}{2} > \frac{7}{2}$ $\boxed{\text{Range of } f(x): x : x > 7/2}$

Good!

Correct solution:

$f(x) = \frac{e^x+7}{2}$ find $f^{-1}(x)$.

$f(x) = \frac{e^x+7}{2} = y = \frac{e^x+7}{2}$

$x = \frac{e^y+7}{2}$ $2x = e^y + 7$

log both sides Not yet!

$\ln 2x = y + 7$

$-7 \quad -7$

$\ln(2x-7) = y$ Here, I had two errors. First I left is as equalling y, instead of what it was asking $f^{-1}(x)$. So, $f^{-1}(x) = \ln 2x - 7$. Second, I forgot the parenthesis around $2x-7$, making it very wrong. Altogether it was very careless + forgetful mistakes! $\boxed{f^{-1}(x) = \ln(2x-7)}$

Supercorrection Four-Point Form Name:

#11b Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 0 /4 Supercorrection 4 /4 Looking back on these, I realize how important it was to know my log rules (11b/11c) For $\log_b a$ trying to get some part of it to equal $\log_a b$ (because I had a value for it: 2.3219) The rule: #8, Base change Formula would have been very helpful. This rule states: $\log_a M = \frac{\log_b M}{\log_b a}$ where b = any legit. base. My biggest error in both b/c was not utilizing what I already knew + referring back to what I was given.

Correct solution:

b. $\log_b a$ $\log_a b = 2.3219$ → Property 8: Base change Formula $\log_a M = \frac{\log_b M}{\log_b a}$ where b can be any legitimate base.

Nice!

This can be written as:

$\frac{\log_a a}{\log_a b}$ ← Another rule → $\log_a a$ = property #2 = prob. #1a. on test = 1. ($\log_a a = 1$) Then I know $\log_a b = 2.3219$ so I have

$\log_a b$ ← I know this value = 2.3219

$\frac{1}{2.3219}$

I originally had $\frac{\log_a a}{\log_a b}$ but by making my base a, I can solve because I know the value of $\log_a b$.

#12 Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 0 /4 Supercorrection 4 /4 To solve this, it would have been helpful to remember multiplying a fraction by its reciprocal is equal to 1. In doing this, it cancels out the first part of the equation making $x = 8^{4/3}$ This was a rather pensive problem; there are so many ways to go about it, it almost gets more confusing.

Correct solution:

$x^{3/4} = 8$ $8 = 2^3$

$(x^{1/4})^3 = 2^3$

$x^{1/4} = 2$ (ignore the 3's)

$(x^{1/4} = 2 \cdot 1)^4$

$x^1 = 2^4$

$\boxed{x = 16}$

put each side to the 4th (could have done this earlier)

$[x^{3/4}]^{4/3} = [8]^{4/3}$

$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$

So $x = 8^{4/3}$

8 can be written as (2^3) so

$(2^3)^{4/3} = 2^{12/3} = 2^4 = 16$

This way is much simpler and easier than the previous way and very fluid. Once I get rid of the x's fraction by multiplying by its reciprocal, the rest I can do.

Supercorrection Four-Point Form Name:______________

#13c Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 3 /4 Supercorrection 4 /4

p. 487 Property 3 + Property 4 The explanation above these two properties is a very good one that I'm pretty sure I never read ~~before~~! If f and g are inverse functions, then $g[f(x)] = x$ and $f[g(x)] = x$. If $f(x) = a^x$ and $g(x) = \log_a x$ then $\log_a(a^x) = a^{\log_a x} = x$
Prop. 3: $\log_a(a^x) = x$ Prop. 4 $a^{\log_a x} = x$. This explains the problem through and through. I know there inverses because $g[h(x)] = x$ and $h[g(x)] = x$. They follow the rules of prop. 3 and 4.

Correct solution:

a. $g(h(x))$

$g[h(x) = 7^x]$

$g(7^x) = \log_7 7^x$

$= x \log_7 7 = \boxed{x}$

b. $h(g(x))$

$h[g(x) = \log_7 x]$

$h(\log_7 x) = 7^{\log_7 x}$

$= \boxed{x}$

What does this mean??

$\rightarrow$ They're inverses

#11 c Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original 3 /4 Supercorrection 4 /4

This would have also been very helpful to know rules. I just forgot on this one that I could write $\log_a ab$ as $\log_a a + \log_a b$ That part being a value I had. Its property #5, which I actually got right on question #1, e. I must not have been paying very close attention, when I had 4 properties already correctly identified. If only I had looked back on my test, everything I needed to know was right there. Good!

Correct solution:

$\log_a b = 2.3219 \rightarrow$ I need to re-arrange $\log_a ab$ into a form involving $\log_a b$ because I have a known variable if I do that (2.3219) so, back to property #5, or problem #1, e on this test: $\log_a xy = \log_a x + \log_a y$. so I can then manipulate my $\log_a ab$ using this rule into $\rightarrow \log_a a + \log_a b$ $\rightarrow$ familiar! Now I know that $\log_a b = 2.3219$ so I have $\log_a a + 2.3219$. Now again referring back to problem #1, this time a. I find the rule (property #2): $\log_a a = 1$ (exactly identical to the log in my problem.) This leaves me with $1 + 2.3219 = \boxed{3.3219}$

$\frac{83.5}{104}$ ≈ 80% I know you understand logs so much better now! Well done.

ALGEBRA 2 HON Name:____________________

A Finely Crafted O'Brien Unit 5 Test

Calculator Section: You may use a calculator. Show all work and circle your answer. Use your time wisely; you will be able to earn additional credit after the timed portion of the test by completing Supercorrections. When you finish, put away your calculator and you come up to get the non-calculator part- you may continue to work on both sections without your calculator.

1. In the following statements, $a > 0$, $a \neq 1$ and $x > 0$, $y > 0$. Determine whether each is true or false.

a. $\log_a a = 1$

$\boxed{\text{True}}$ $a^1 = a$

b. $\log_a 1 = a$

$a^a = 1$ $\boxed{\text{False}}$

c. $\log_a 0 = 1$

$a^1 = 0$ $\boxed{\text{False}}$

d. $\log_a 1 = 0$

$a^0 = 1$ $\boxed{\text{True}}$

e. $\log_a(xy) = \log_a x + \log_a y$

$\boxed{\text{True}}$ $3 + 2 = 5$ $2^y = 2$ $\log_2 6 = 2^y = 6$

f. $\log_a(x+y) = \log_a x + \log_a y$

$\log_2 5 = 2^y = 5$ $\boxed{\text{False}}$ $3+2=5$

g. $\log_a(x+y) = \log_a x \cdot \log_a y$

$\boxed{\text{False}}$

h. $\log_a xy = \log_a x \cdot \log_a y$

$= 5$ $3 \cdot 2$ $\boxed{\text{False}}$

i. $\log_a x^y = y \log_a x$

$\boxed{\text{True}}$

j. $\log_a x^y = (\log_a x)^y$

$\boxed{\text{False}}$

2. Find the smallest value of x for which $3^x \geq 1{,}000{,}000$.

8

$3^x \geq 1{,}000{,}000$

$\log 3^x = \log 1000000$

$\frac{x \log 3}{\log 3} = \frac{\log 1000000}{\log 3}$

Gives 1000000.005

$x = 12.57541965\,49$

So close!

12.575419649 Gives 1000000.004

3. The number of bacteria B in a culture increases according to the equation $B = B_0 e^{kt}$. There were 400 bacteria at time $t = 0$ and 900 bacteria at time $t = 4$ hours. Will there be 1900 bacteria after 12 hours? If not, how many will there be?

7.5

$B = B_0 e^{kt}$

$400\ B = t(0)$

$900\ B = t(4)$

Close! Rounding error.

$400 = 400 e^{K(0)}$

$B_0 = 400$

$\frac{900}{400} = \frac{400 e^{K(4)}}{400}$

$2.25 = e^{4K}$

$\frac{\ln 2.25}{4} = \frac{4K}{4}$

$K = .203$

$\cancel{\frac{\ln 4.75}{12} = K}$

$\cancel{K = .13}$

$\frac{\ln 9/4}{4} = K$

$\frac{1900}{400} = \frac{400 e^{12K}}{400}$

$4.75 = e^{12K}$

$\cancel{\ln 4.75 = 12K}$

$\boxed{\approx 4556}$

No, there ~~will~~ be ≈ 4571 bacteria

$B = 400 e^{.203(12)}$

$B = 4570.896$

4. At the right is a "solution" to the equation $100 = 18e^{4k}$.

a. Check the answer $k \approx 0.398$ back in the equation $100 = 18e^{4k}$ and show that it doesn't work.

$100 = 18e^{4k}$

$\ln 100 = \ln(18e^{4k})$

$\ln 100 = \ln 18 \ln(e^{4k})$

$\frac{\ln 100}{\ln 18} = \ln(e^{4k})$

$\frac{\ln 100}{\ln 18} = 4k$

$\frac{\ln 100}{4 \ln 18} = k$

$k \approx 0.398$

$100 = 18e^{4(.398)}$

$\boxed{100 \neq 88.4442}$

b. Circle the error in the "solution" and give a correct solution below.

$\frac{100}{18} = \frac{18e^{4k}}{18}$

$\frac{\ln \frac{100}{18}}{4} = \frac{4k}{4}$

$k = \frac{\ln\left(\frac{100}{18}\right)}{4}$

$k = .4287$

$18e^{4(.4287)} = 100.0001572$

plug in

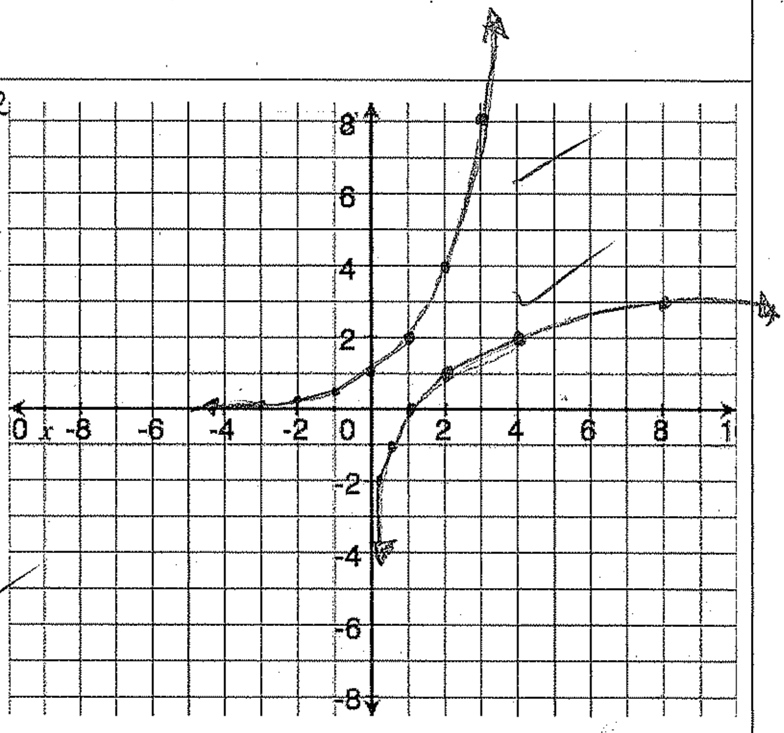
7

5.

a. Make a table and accurately graph $y = 2^x$. - no space

x	-2	-1	0	1	2	3	4
y	.25	.5	1	2	4	8	16

-3
.125



b. Make a table of values for the inverse of $y = 2^x$.

x	.25	.5	1	2	4	8	16
y	-2	-1	0	1	2	3	4

c. Sketch the graph of the inverse of $y = 2^x$.
(Label which graph is which.)

d. Write the equation of the inverse of $y = 2^x$ in $y =$ form.

$y = 2^x$

$\boxed{x = 2^y}$

$\boxed{y = \log_2 x}$

$x = 2^y$

$\log_2 x = y$

6. Use the properties of logs to write the expression as a sum, <u>difference</u>, and/or multiple of logs. <u>Simplify</u> where possible.

$$\log \frac{100\sqrt{y}}{x^2}$$

$= \log 100\sqrt{y} - \log x^2$

$= 2+\sqrt{y} - \log x^2$

$\log 100 + \log \sqrt{y}$

$\log 100\sqrt{y} \cdot \log x^2$

$\log 2\sqrt{y}$

$\boxed{= 2 + \frac{1}{2}\log y - 2\log x}$

6

7. Write the expression as the logarithm of a single quantity.

$$\ln 3 + \frac{1}{3}\ln(4 - x^2) - \ln x$$

$\frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$

$\ln 3 + \frac{1}{3}\ln 4 - \frac{1}{3}\ln x^2 - \ln x$

$\ln 3 + \ln 4^{1/3} - \ln x^{2/3} - \ln x$

$\boxed{= \ln \frac{3\sqrt[3]{4-x^2}}{x}}$

3

8. Molly needs to find the range of $f(x) = \frac{e^x + 7}{2}$. She has a brilliant idea that she can use the inverse of $f(x)$ in a clever way to do this. Find $f^{-1}(x)$ and use it to find the range of $f(x)$.

$f(x) = \frac{e^x + 7}{2} = y = \frac{e^x + 7}{2}$

Range = $\mathbb{R}$ #'s

$x = \frac{e^y + 7}{2} = 2x = e^y + 7$

$\ln 2x = y + 7$
$-7 \quad -7$

$f^{-1}(x) = \ln(2x - 7)$

$y = \ln(2x - 7)$

$2x - 7 > 0$

Range is $f(x) > 7/2$

Close!

6

9. Select values for a and b, with $a > b$, such that $\log_b a$ is less than zero. Justify your choice for a and b.

$\log_{b\,2} a^4 < 0 \qquad b^y = a$

$a > b$

$a = 3$
$b = 2$

no real solution

$a = 4,\ b = 1/2$

5

NOPE! $\log_a 4+\log_a(4-2)=\log_a(4+4)$

$\log_a 8=\log_a 8$ ✓

10. Solve for x. plug back $\log_a(-1)$

$\log_a x+\log_a(x-2)=\log_a(x+4)$

$\boxed{x=4}$

$x=+4$ or -1

6

$\log_a x^2-2x=\log_a(x+4)$

$\log_a x^2-3x-\log_a 4=0$

$x^2-2x=x+4$

$x^2-3x-4=0$

$(x+1)(x-4)$

$\frac{2.3219}{2}$ 4.6438

$a^{2.3219}=b$

11. Given that $\log_a b\approx 2.3219$, approximate

a. $\log_a \frac{1}{b^2}$

b. $\log_b a$ $\frac{\log_a a}{\log_a b}$

c. $\log_a ab$ $=\log_a a+\log_a b$

$a^1=ab$ $=1+2.3219$

6

$=\log_a 1-\log_a b^2$

$0-$ $\log_a b^2=2\log_a b$

$x=-4.6438$

$\boxed{\frac{1}{2.3219}}$

$=\boxed{3.3219}$

12. Solve for x.

$x^{\frac{3}{4}}=8$

6

$\log_x 8=3/4$

$x^{3/4}=2^{(3)}$

multiply 3/4 by what to get 3

$\frac{3}{4}x=3$

$\log_x 8=.75$

$\boxed{x=4}$ $\boxed{x=16}$

$x=\frac{12}{3}$ $x=4$

13. Suppose $g(x)=\log_7 x$ and $h(x)=7^x$.

$x=2^3$

a. Find $g(h(x))$.

b. Find $h(g(x))$.

$g[h(x)=7^x]$

$h[g(x)=\log_7 x]$

7

$g(7^x)=\log_7 7^x$

$h(\log_7 x)=7^{\log_7 x}$

$=x\log_7 7=x$ ✓

$=x$ ✓

c. What is the relationship between the functions g and h?

They both equal x So, ...

Close! $\boxed{\text{Inverses}}$!

Bonus: Choose values for a, b, and c so that the equation below is true. Justify your choice of a, b, and c.

$\log_c[\log_a(\log_b c)]=0$

$\log_{28} 7$

$\frac{\log_{10} 7}{\log_{10} 28}$

$\log_a a$

$\log_b b$